The method used to calculate the ionospheric electron density is based on statistical inversion theory. Both the measurements and the unknown electron densities are treated as random variables, and the algorithm gives the most probable values of electron density. (For a more fully explanation of the used method see *Nygrén et al.,* 1997.)

The phase difference of received 150 MHz and 400 MHz signals is proportional to the integral of electron density along the ray from satellite to the receiver, but it contains one unknown constant for each receiver, because the phase difference at the start time of the measurement is known only up to 2π. These unknown phase constants are also treated as random variables in the algorith.

The vertical plane above the receiver chain is divided into an annular grid, and the unknowns are the electron densities at the grid points. When we suppose that electron densities are constant in lines perpendicular to that plane, the measured phase values are linear combinations of the unknown electron values and phase constants. If all measurements are collected into vector $\mathbf{m}$ and all unknowns into vector $\mathbf{x}$, we can write

$$\mathbf{m} = \mathbf{A}_\mathrm{m} \cdot \mathbf{x} + \varepsilon_\mathrm{m}, \tag{1}$$

where $\mathbf{A}_\mathrm{m}$ is a matrix of linear coefficients and ε_m is a vector containing the measurement errors.

In the ionospheric tomography the equation (1) is unstable and no meaningfull results can be obtained without some kind of regularization.

The regularization is made by giving for all neighboring grid points fictitious measurements

$$0 = x_i - x_j + \varepsilon_r^{ij} \tag{2}$$

where ε_r^{ij} are regularization errors with variances σ_r^{ij}, which are proportional to a vertical regularization profile. This allows larger variation of the electron density in the altitudes, where the profile has large values. All the equations (2) can be combined together with equation (1) to make a single matrix equation

$$\begin{pmatrix} \mathbf{m} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_\mathrm{m} \\ \mathbf{A}_\mathrm{r} \end{pmatrix} \cdot \mathbf{x} + \begin{pmatrix} \varepsilon_\mathrm{m} \\ \varepsilon_\mathrm{r} \end{pmatrix}. \tag{3}$$

The regularization method can be modified so that it forces the electron densities to zero at very low and/or high altitudes. In equation (3) one can add an extra line for each corresponding grid point, which puts the electron density to zero with a very small error.

The electron densities are calculated by using two different regularizations. In the first case the regularization profile is the same chapman layer with 280 km peak altitude 280 km and scale height 140 km for all measurements. In this case the electron density is forced to zero both at the ground level and satellite height. In the second case the regularization profile is the time dependent layer given by IRI (International Reference Ionosphere) -model above the Kokkola receiver site, shifted downwards 50 km. In this case the electron density is forced to zero only at the ground level.

References

Nygrén, T., M. Markkanen, M. Lehtinen, E. D. Tereshchenko, B. Z. Khudukon, Stochastic inversion in ionospheric radiotomography, *Radio Science*, **32,** 2359-2372, 1997.